CHAPTER

16

Gardner's Equation for Water Movement to Plant Roots

W.R. Gardner was one of the first people to model analytically water movement to plant roots. His solution is widely cited today and known by any plant or soil scientist modeling water movement to plant roots. Therefore, it is important for us to understand this basic work. (For a biography of Gardner, see the Appendix, Section 16.10.)

16.1 DESCRIPTION OF THE EQUATION

In his paper, Gardner (1960) solved the flow equation to determine water movement to a plant root. His solution was the first (Green et al., 2006) and followed up on the outline of the problem by Philip (1957). The flow equation for a single root in an infinite, two-dimensional medium is

$$\partial\theta/\partial t = (1/r)(\partial/\partial r)[rD(\partial\theta/\partial r)] \tag{16.1}$$

where θ is the water content of the soil on a volumetric basis, D is the diffusivity, t is the time, and r is the radial distance from the axis of the root. The solution to Eqn (16.1), subject to boundary conditions that Gardner (1960) defines, is given by Carslaw and Jaeger (1959) (Gardner cites a 1947 reprint of the first edition of Carslaw and Jaeger published in 1946):

$$\tau - \tau_0 = (q/4\pi k)\left[\ln\left(4Dt/a^2\right) - \gamma\right], \tag{16.2}$$

where $\gamma = 0.57722\ldots$ is Euler's constant, a is the radius of the root, k is the unsaturated hydraulic conductivity of the soil, q is the rate of water uptake by the root, D is the diffusivity of the soil, τ is the soil suction, and τ_0 is the suction in the soil for initial conditions. We are interested in the difference in suction required at the boundary between the plant root and the (bulk) soil to maintain a constant rate of water movement to the plant.

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(We will refer to the soil at some distance from the root as the "bulk" soil.) The solution to Eqn (16.2) assumes a constant k and D. Because root diameters are small, it is possible to consider the root as a line source, or in this case, sink, of strength q per unit length for which the solution of Eqn (16.1) also is given by Carslaw and Jaeger (1959):

$$\tau - \tau_0 = (q/4\pi k)\left[\ln\left(4Dt/r^2\right) - \gamma\right]. \tag{16.3}$$

When $r = a$, Eqns (16.2) and (16.3) are the same.

Gardner pointed out that study of Eqn (16.3) shows that a large part of the water being taken up by the roots comes from some distance from the roots. It is instructive to compare Eqn (16.2) with the steady-state solution for flow in a hollow cylinder:

$$\tau - \tau_0 = (q/4\pi k)\left[\ln\left(b^2/a^2\right)\right], \tag{16.4}$$

where τ_0 is now the suction at the outer radius of the cylinder $r = b$ and τ is the suction at the inner radius $r = a$. If we take $b = 2(Dt)^{½}$, Eqn (16.4) becomes identical with Eqn (16.2) except for the constant term γ. γ is relatively small compared with the logarithmic term, so that the distribution of suction in the transient case is not very different from that in the steady-state case, with all the water coming from a distance $b = 2(Dt)^{½}$. The maximum radius b is limited by the density of roots and can be taken as one-half the average distance between neighboring roots.

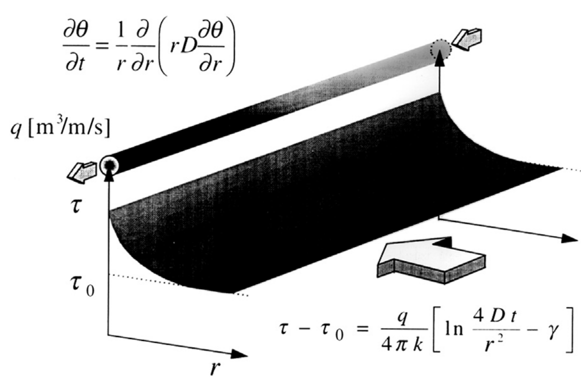

FIGURE 16.1 For the gravity-free, linearized form of the Richards equation with a radial coordinate (equation at top of figure), Gardner (1960) solved for the field of suction surrounding a root of sink strength q, in terms of the suction τ, distance r, and time t, given the soil's hydraulic characteristics (the unsaturated hydraulic conductivity k and diffusivity D). γ is Euler's constant. *From Clothier and Green (1997). This material is used by permission of Lippincott Williams & Wilkins, A Wolters Kluwer Company: Hagerstown, Maryland; and Brent E. Clothier.*

Using the modern terminology of matric potential instead of the old terminology of soil suction, Eqn (16.4) becomes (Baver et al., 1972, p. 404):

$$\Psi_b - \Psi_a = (q/4\pi k)\left[\ln\left(b^2/a^2\right)\right], \tag{16.5}$$

where Ψ_b (bars or MPa) is the matric potential midway between roots and is that which could be measured by any finite-sized measuring device, Ψ_a (bars or MPa) is the matric potential at the plant root–soil boundary, q is the volume of water taken up per unit length of root per unit time (cm^3/cm/s or mL/cm/day), and k is the unsaturated hydraulic conductivity of the soil (cm^2/s-bar). Gardner (1960) neglects any contact resistance at the soil–root interface. Figure 16.1 from Clothier and Green (1997) provides a picture of Gardner's model.

16.2 ASSUMPTIONS

Gardner (1960) made several assumptions in deriving Eqn (16.4) (or Eqn (16.5)), as follows:

1. The roots are infinitely long cylinders and a distance $2b$ apart.
2. The roots have a uniform radius $= a$.
3. There is uniform water absorption along the root. (Kramer (1969, p. 179) reports that, as expected, the highest rate of water entry into roots is found in root hairs and unsuberized roots and the lowest rate of water entry into roots occurs in suberized woody roots. However, even suberized roots do have a low rate of water entry. They are not completely impermeable to water.)
4. Water moves in a radial direction only (gravity-free movement).
5. There is a uniform value for the initial soil water content, θ_0, and it corresponds to an initial matric potential in the bulk soil, Ψ_0 for Ψ_b.

16.3 VALUES FOR THE RATE OF WATER UPTAKE

What might values of q be? The uptake of water by a young root 1 mm in diameter is usually 0.1–0.5 mL/day/cm of root length (e.g., see Nobel, 1974, p. 389). If we take an intermediate value of 0.3 mL/day/cm of root, this corresponds to:

$$q = \left(0.3\ \mathrm{cm}^3/\mathrm{day}\right)(1\ \mathrm{day}/86{,}400\ \mathrm{s}) = 3.5 \times 10^{-6}\ \mathrm{cm}^3/\mathrm{s/cm\ of\ root}.$$

As an aside, we can compare this q to values of flux (J_v) reported by Nobel (1974, p. 393) for transpiration rates of diffuse-porous and ring-porous wood. This q of 3.5×10^{-6} cm^3/s occurs over a root surface area

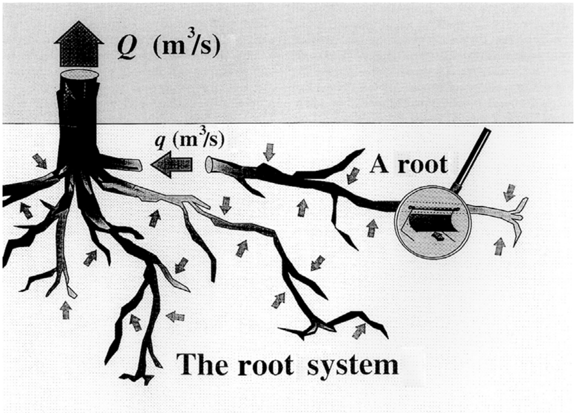

FIGURE 16.2 Gardner's (1960) model of uptake describes the local field of flow in the soil that is required to supply active root segments with an internal flux of sap flow q (magnifying glass). The anastomosis of these spatially distributed fluxes in the roots forms the plant's total transpiration Q. *From Clothier and Green (1997). This material is used by permission of Lippincott Williams & Wilkins, A Wolters Kluwer Company: Hagerstown, Maryland; and Brent E. Clothier.*

of $2\pi rl$, which is the surface area of a cylinder with radius r and length l. So, we have

$$(2\pi)(0.05 \text{ cm})(1 \text{ cm}) = 0.31 \text{ cm}^2.$$

Thus J_v or flux at the root surface is

$$\left(3.5 \times 10^{-6} \text{ cm}^3/\text{s}\right)/\left(0.31 \text{ cm}^2\right) = 1.1 \times 10^{-5} \text{ cm/s}.$$

Nobel assumed a J_v for diffuse-porous wood of 0.1 cm/s and for ring-porous wood of 1 cm/s. The J_v for the root is four orders of magnitude less than that for the diffuse-porous tree and five orders of magnitude less than that for the ring-porous tree. We note that the q (or J_v) being considered by Gardner is that to a root, which is much less than the total flux that accumulates in the shoot from numerous roots penetrating the soil and funneling all the water into one stem (Figure 16.2). Water absorption by plant roots occurs at a slower rate than movement of water through the open vertical tubes of the xylem vessels in the stems of plants.

16.4 EXAMPLES

Let us follow an example given by Nobel (1974, p. 389). Let us assume that at $b = 1.05$ cm, $\Psi_b = -2$ bars, which we can measure with a device such as a thermocouple psychrometer. (Note: We could not measure

2 bars tension with a tensiometer because it is beyond the tensiometer range.) Let us assume that $k = 1 \times 10^{-6}$ cm^2/s bar, which might apply to a loam of moderately low water content.

Using Eqn (16.5):

$$-2 \text{ bars} - (\Psi_a) = \left\{3.5 \times 10^{-6} \text{ cm}^3/\text{cm s}/(4\pi)\left(1 \times 10^{-6} \text{ cm}^2/\text{s bar}\right)\right\} \times \left\{\ln\left[(1.05 \text{ cm})^2/(0.05 \text{ cm})^2\right]\right\}$$

Solving, we get $\Psi_a = -3.7$ bars or about -4 bars. Thus, the matric potential drops about 2 bars across a distance of 1.05 cm in the soil next to the root.

We can consider an osmotic (solute) component in the soil that is constant throughout the soil. As we saw in Chapter 4 (Eqn (4.3)), the equation for total water potential, Ψ, is

$$\Psi = \Psi_m + \Psi_s + \Psi_g + \Psi_p,$$

where Ψ_m is the matric potential, Ψ_s is the osmotic (solute) potential, Ψ_g is the gravitational potential, and Ψ_p is the pressure potential. But the last two terms on the right-hand side drop out because water is moving laterally in the soil (gravity stays the same) and we are dealing with unsaturated conditions, so $\Psi_p = 0$. The Ψ_s is additive to the Ψ_m. Let us assume $\Psi_s = -1$ bar.

For Ψ_a, we have:

$$\Psi_m + \Psi_s = -4 \text{ bars} + (-1 \text{ bar}) = -5 \text{ bars}.$$

For Ψ_b, we have:

$$\Psi_m + \Psi_s = -2 \text{ bars} + (-1 \text{ bar}) = -3 \text{ bars}.$$

The difference in matric potential between the root–soil boundary and the bulk soil at 1.05 cm away from the root remains the same (2 bars) as in the nonsaline case.

16.5 EFFECT OF WET AND DRY SOIL

Gardner (1960) applied his equation to different situations. Figure 16.3 shows the matric potential at the root (Ψ_a) as a function of the distance from the root, when the matric potential in the bulk soil (Ψ_b) is -5 or -15 bars and the rate of uptake is 0.1 mL/cm/day. At -5 bars matric potential, the gradient is very small, except right at the root, so that the matric potential is virtually uniform throughout the soil. When $\Psi_b = -15$ bars, a large gradient is required for the same q, because of the lower hydraulic conductivity.

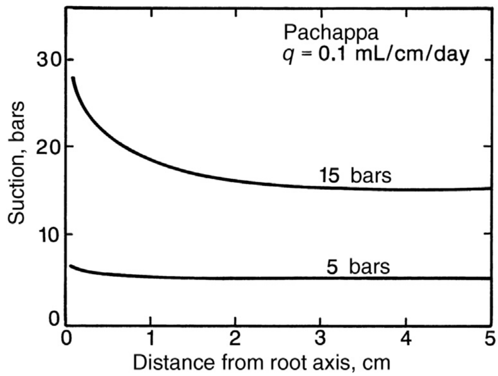

FIGURE 16.3 A solution to Gardner's equation. It shows the suction at the plant root as a function of the distance from the root when the suction in the bulk soil is 5 or 15 bars. In current terminology, suction (positive value) is now called matric potential (negative value). *From Gardner (1960). This material is used by permission of Lippincott Williams & Wilkins, A Wolters Kluwer Company: Hagerstown, Maryland; and Wilford R. Gardner.*

16.6 EFFECT OF ROOT RADIUS

Figure 16.4 shows the effect of root radius. On the ordinate is the relative value of $\Delta\tau$ (difference in matric potential between the bulk soil and soil–root boundary). The scale on the ordinate is arbitrary because the actual matric potential would depend on the initial matric potential, q, and k. The figure shows that root radius is not extremely important. A tenfold increase in root radius can bring about only approximately a twofold decrease in the difference in matric potential between the bulk soil and soil–root boundary.

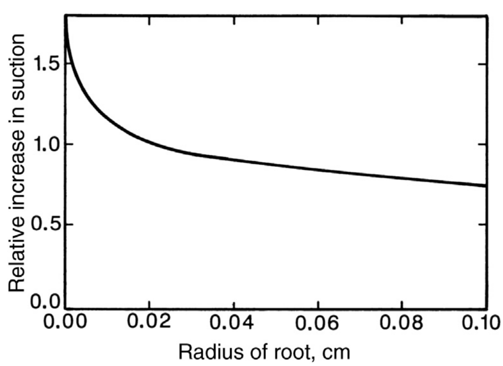

FIGURE 16.4 A solution to Gardner's equation. It shows the relative increase in suction at the plant root as a function of the root radius. *From Gardner (1960). This material is used by permission of Lippincott Williams & Wilkins, A Wolters Kluwer Company: Hagerstown, Maryland; and Wilford R. Gardner.*

16.7 COMPARISON OF MATRIC POTENTIAL AT ROOT AND IN SOIL FOR DIFFERENT RATES OF WATER UPTAKE

Figure 16.5 shows suction at the plant root plotted as a function of the suction in the bulk soil for a Pachappa sandy loam for three different rates of water uptake. Except for unusual circumstances, the lowest rate indicated (0.05 cm^2/day) is probably more nearly that which occurs in nature and may, in fact, be high for a fully developed root system. The value of $q = 0.1$ cm^2/day is consistent with data of Ogata et al. (1960) for alfalfa. The figure shows that the suction at the root does not exceed the bulk soil suction appreciably until the suction difference is a few bars. When the bulk soil suction is as high as 15 bars, the suction at the root must be 30 or 40 bars to maintain a given value of q. The figure shows that, over a short distance, there is little difference in the suction between the root and bulk soil. But as the soil becomes dry, the difference becomes large in order to maintain a constant q. (Figure 16.5 from the original 1960 paper has been reproduced by Baver et al., 1972, p. 405.)

The main point of Figure 16.5 is that over short distances there is little difference in suction between the root and soil in the vicinity of the root until the soil becomes very dry. Even then, the flux tends to decrease markedly due to plant wilting. Gardner's analysis shows that in many cases one can probably assume that the roots are at very nearly the same potential as the surrounding soil (Baver et al., 1972, p. 404).

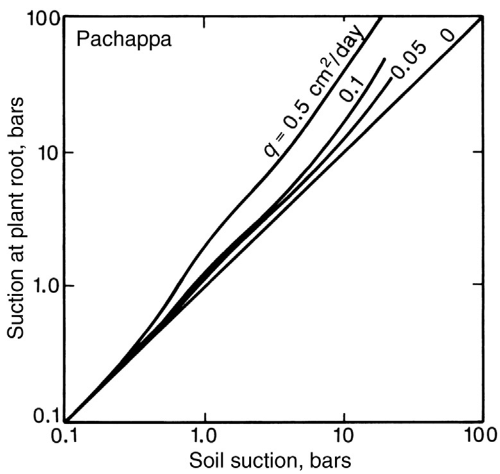

FIGURE 16.5 Suction at the root–soil boundary as a function of the bulk soil suction for different rates of water uptake q. *From Gardner (1960). This material is used by permission of Lippincott Williams & Wilkins, A Wolters Kluwer Company: Hagerstown, Maryland; and Wilford R. Gardner.*

16.8 EFFECT OF ROOT DISTRIBUTION ON WILTING

The rate of uptake of water per unit length of root is proportional to the total transpiration rate, and inversely proportional to the length of the root system (Gardner, 1960, p. 68). Assuming a given rate of transpiration, the more extensive the root system the lower is the rate of uptake per unit length of root. The uptake rate q thus depends on the transpiration rate and the extent of the root system. To study the effect of transpiration rate and extent of root system on the wilting of a plant, Gardner (1960) assumed that wilting occurs when the suction in the plant root is above some value, say 20 bars. In Figure 16.6 the average soil suction that results in a suction of 20 bars in the plant root is plotted as a function of q. This is, then, a plot of the soil suction at the wilting point as a function of q. For low rates of uptake, the wilting suction of the soil is very nearly the suction in the plant. For high values of q, large differences between the two are possible (Gardner, 1960).

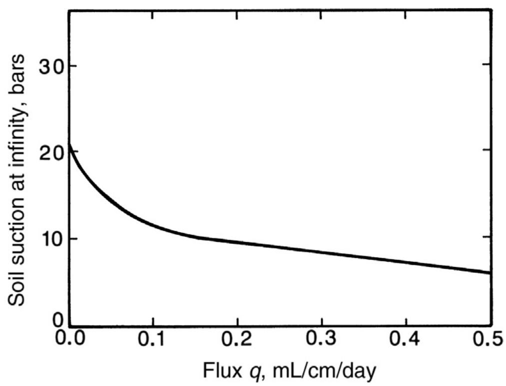

FIGURE 16.6 The soil suction at a distance from the root, when suction at the plant root is 20 bars, plotted as a function of water uptake rate. *From Gardner (1960). This material is used by permission of Lippincott Williams & Wilkins, A Wolters Kluwer Company: Hagerstown, Maryland; and Wilford R. Gardner.*

16.9 FINAL COMMENT

Gardner said, when his 1960 paper became a citation classic (Institute for Scientific Information, 1985), "I believe this paper has been so frequently cited because the approach is essentially the same that all computer models of plant water uptake now follow". Clothier and Green (1997) and Green et al. (2006) discuss the incorporation of Gardner's theoretical ideas into the many simulation models of root zone function that have followed.

16.10 APPENDIX: BIOGRAPHY OF WILDFORD GARDNER

Wilford Robert Gardner, a physicist and educator, was born on October 19, 1925, in Logan, Utah, the son of Robert and Nellie (Barker) Gardner. After serving in the U.S. Army during World War II (1943–1946) (Marquis Who's Who, 2003), he got his B.S. degree from Utah State University in 1949 and his M.S. degree from Iowa State University in 1951 under the direction of Don Kirkham. They did the pioneering work on the neutron probe, which is now widely used to monitor soil water content (Gardner and Kirkham, 1952). Gardner got his Ph.D. degree at Iowa State University in 1953 under the guidance of Gordon C. Danielson. Gardner's uncle, Willard Gardner, brother to his father Robert, was the famous soil physicist at Utah State College (as it was known then). Willard Gardner is considered to be the "Father of Soil Physics in America." Gardner's cousin, Walter H. Gardner, the son of Willard, spent his career as a soil physicist at Washington State University in Pullman.

After obtaining his Ph.D., Wilford joined the U.S. Salinity Laboratory in Riverside, California. There, Gardner did seminal work deriving equations for flow of water under unsaturated conditions (Gardner, 1958; Gardner and Mayhugh, 1958), which are used worldwide by mathematical soil physicists today. This theoretical work has resulted in a soil being named after him, the "Gardner soil" (Kirkham and Powers, 1972, p. 275; Kacimov, 2007; Youngs and Kacimov, 2007). At the U.S. Salinity Laboratory he also wrote his important paper on water movement to a plant root (Gardner, 1960).

In 1966, Gardner moved to the University of Wisconsin, where he was a professor in the Soil Science Department. His students there included M.B. Kirkham, Frank N. Dalton, David B. Lesczynski (who died of a heart attack on August 4, 2001, at the age of 55 years), William A. Jury, William N. Herkelrath, John H. Knight, Don F. Yule, and K. John McAneney. In 1980, he moved from the University of Wisconsin to become head of the Department of Soil and Water Science at the University of Arizona, Tucson. In 1987, he accepted the position of dean of the College of Natural Resources at the University of California, Berkeley. In addition to being dean, he was associate director of the California Agricultural Experiment Station and professor of soil physics in the Departments of Plant & Soil Biology and Forestry & Resource Management. He was one of the few scientists who was both an outstanding researcher and an able administrator. In 1994, he became dean emeritus, and from 1995 until his death on May 20, 2011, from Parkinson's disease (Kirkham, 2011), he was adjunct professor at Utah State University.

Gardner received many recognitions. In 1959, he was National Science Foundation (NSF) Senior Fellow and in 1971–1972, he was a Fulbright Fellow. In 1972, he was recipient of an Honorary Faculty Award at the University of Ghent, Belgium. In 1983, he was elected to the National Academy of Sciences. He received the Centennial Alumnus Award at Utah State University in 1986. He was Fellow of the American Association for the Advancement of Science, the American Society of Agronomy, and the Soil Science Society of America. He was President of the Soil Science Society of America in 1990 and received its Research Award in 1962. He was an Honorary Member of the International Union of Soil Science (Marquis Who's Who, 2003). In 2002, he received an honorary doctor's degree from The Ohio State University.

He was a strong believer in basic research. At the annual William H. Pierre Lecture, which he gave at Iowa State University on April 1, 1986, he ended his talk with the admonition that basic research should be supported, even if it does not accomplish immediate practical results for specific, desired goals.

Gardner married Marjorie Louise Cole, the granddaughter of Willard Gardner, on June 9, 1949. They had three children, Patricia (died in 2008), Robert, and Caroline. He wrote his memoirs in two volumes, both self published: *My War* (304 pages; undated but distributed in 2009) and *Memoirs of a 'A Fair to Middlin' Physicist* (454 pages; 2009).

References

Baver, L.D., Gardner, W.H., Gardner, W.R., 1972. Soil Physics, fourth ed. Wiley, New York.

Carslaw, H.S., Jaeger, J.C., 1959. Conduction of Heat in Solids, second ed. Clarendon Press, Oxford (First ed., 1946).

Clothier, B.E., Green, S.R., 1997. Roots: the big movers of water and chemical in soil. Soil Sci. 162, 534–543.

Gardner, W.R., 1958. Some steady-state solutions of the unsaturated moisture flow equation with application to evaporation from a water table. Soil Sci. 84, 228–232.

Gardner, W.R., 1960. Dynamic aspects of water availability to plants. Soil Sci. 89, 63–73.

Gardner, W.R., Kirkham, D., 1952. Determination of soil moisture by neutron scattering. Soil Sci. 73, 391–401.

Gardner, W.R., Mayhugh, M.S., 1958. Solutions and tests of the diffusion equation for the movement of water in soil. Soil Sci. Soc. Am. Proc. 22, 197–201.

Green, S.R., Kirkham, M.B., Clothier, B.E., 2006. Root uptake and transpiration: from measurements and models to sustainable irrigation. Agr. Water Manage. 86, 165–176. http://dx.doi.org/10.1016/j.agwat.2006.06.008.

Institute for Scientific Information, Gardner, W.R., 1985. Dynamic aspects of water availability to plants. Soil Sci. 89, 63–73. This Week's Citation Classic. Current Contents, Agr. Biol. Environ Sci. 16(35); 20.

Kacimov, A., 2007. Dipole-generated unsaturated flow in Gardner soils. Vadose Zone J. 6, 168–174. http://dx.doi.org/10.2136/vzj2005.0141.

Kirkham, D., Powers, W.L., 1972. Advanced Soil Physics. Wiley, New York.

Kirkham, M.B., 2011. Vale—Wilford R. Gardner (October 19, 1925—May 20, 2011). Eulogy. In: A Newsletter about Water in the Soil—Plant—Atmosphere System, vol. 109. WISPAS, 10.

Kramer, P.J., 1969. Plant and Soil Water Relationships: A Modern Synthesis. McGraw-Hill Book Co., New York.

Marquis Who's Who, 2003. Who's Who in America, fifty seventh ed. Marquis Who's Who, New Providence, New Jersey.

Nobel, P.S., 1974. Introduction to Biophysical Plant Physiology. W.H. Freeman, San Francisco.

Ogata, G., Richards, L.A., Gardner, W.R., 1960. Transpiration of alfalfa determined from soil water content changes. Soil Sci. 89, 179–182.

Philip, J.R., 1957. The physical principles of soil water movement during the irrigation cycle. In: Proceedings of the Congress of the International Commission on Irrigation and Drainage, Third, Algiers. R. 7, Question 8, 8.125–8.154.

Youngs, E.G., Kacimov, A.R., 2007. Critical tunnel cavities for water exclusion in "Green and Ampt" and "Gardner" soils. Water Resour. Res. 43 (5). http://dx.doi.org/10.1029/2006WR004907. W05501.