CHAPTER

30

Solar Time and Interception of Direct-Beam Solar Radiation

As we saw in Chapter 25, the Sun is the battery that drives processes on earth, including evapotranspiration (Chapter 28). The amount of water lost by evapotranspiration is central to plant–water relations. Another key process driven by interception of the Sun's rays is photosynthesis, which provides the light necessary for the photosynthetic reaction, as follows:

$$6CO_2 + 6H_2O \xrightarrow[\text{Chlorophyll}]{\text{light}} C_6H_{12}O_6 + 6O_2$$

In this reaction, carbon dioxide (CO_2) and water (H_2O) join in the presence of light and chlorophyll to form sugar ($C_6H_{12}O_6$) and oxygen (O_2). Life on earth would not be possible without photosynthesis.

How do plant leaves intercept the rays from the Sun? Before we consider that question, we first need to understand basic principles of astronomy. We begin with the concept of time, which is related to the Sun. We rely on the description of this topic by Abell (1975).

30.1 TIME OF DAY

As we look up to the sky at night, we get the impression that the sky is a great hollow spherical shell with the earth at the center. The early Greeks regarded the sky as just such a *celestial sphere* (Abell, 1975; p. 13). The dictionary (Webster's New World Dictionary of the American Language, 1959) defines the celestial sphere as "the infinite sphere of the heavens hypothecated from the half visible from a point on the earth". The *ecliptic* is the apparent path of the sun on the celestial sphere (Abell, 1975; p. 678). The point on the celestial sphere directly above an observer (defined as opposite the direction of a plumb bob) is the observer's *zenith*. Straight down, 180° from the zenith, is the observer's *nadir*. Halfway between, and 90° from each, is the observer's *horizon* (Abell, 1975; p. 114).

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A *great circle* is any circle on the surface of a sphere (any sphere and not specifically the celestial sphere) whose center is at the center of the sphere (Abell, 1975; p. 113). The earth's equator is a great circle on the earth's surface halfway between the North Pole and the South Pole. The Sun's ecliptic is great circle on the celestial sphere. We can also imagine a series of great circles that pass through the North and South Poles. These circles are called *meridians*. They intersect the equator at right angles. A meridian can be imagined passing through an arbitrary point on the surface of the earth. The great circle passing through the celestial poles and the zenith (and also through the nadir) is called the observer's *celestial meridian* (Abell, 1975; p. 114). This meridian specifies the east–west location of that place. The longitude of the place is the number of degrees, minutes, and seconds of arc along the equator between the meridian passing through the place and the one passing through Greenwich, England, the site of the old Royal Observatory (Abell, 1975; p. 113). The meridian passing through Greenwich, which is a borough of London, also is called the Prime Meridian (Webster's New World Dictionary of the American Language, 1959), and its longitude is $0° 0' 0''$. [For the difficulty in determining longitude, and the person (John Harrison) who finally figured out how to measure it by making a precise clock that worked at sea, see Sobel (2007).] My globe [Cram's Scope-O-Sphere, 12 in (30 cm) World Globe, The George F. Cram Co., Inc., Indianapolis, Indiana] designates the meridian 180° away from the Prime Meridian or Meridian of Greenwich as the *equinoctial colure*. The word colure comes from the Greek *kolouroi*, which literally means dock-tailed (ones) and this word comes from *kolos*, docked, plus *oura*, tail; so named because the "tail" (i.e., the lower part) is always cut off from view by the horizon—at least in Greece and comparable latitudes (Webster's New World Dictionary of the American Language, 1959). *Colure* means either of two imaginary circles of the celestial sphere intersecting each other at right angles at the poles: one passes through the ecliptic at the solstice, the other at the equinox (Webster's New World Dictionary of the American Language, 1959). The *solstice* is either of two points on the Sun's ecliptic at which it is farthest north or farthest south of the equator; it is also the time at which the Sun reaches either of these two points, called the *summer solstice* and the *winter solstice*. The *equinox* is the time when the Sun crosses the equator, making night and day of about equal length in all parts of the earth (Webster's New World Dictionary of the American Language, 1959). We shall discuss the solstice and equinox later in this chapter.

The most obvious coordinate system is based on the horizon and zenith of the observer. Great circles passing through the zenith (*vertical circles*) intersect the horizon at right angles. Imagine a vertical circle through a particular star (Figure 30.1). The *altitude* of that star is the number of degrees along this circle from the horizon up to the star. It is

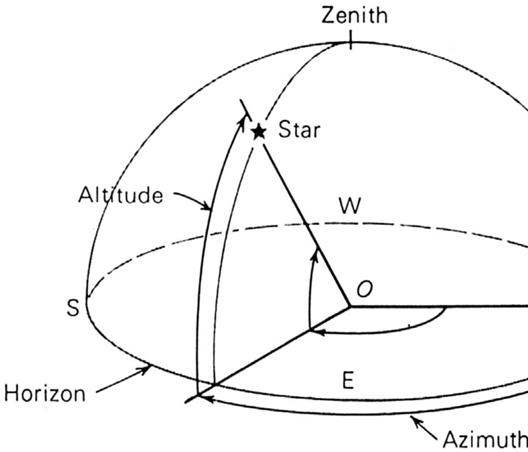

FIGURE 30.1 Altitude and azimuth. *From Abell (1975), p. 115. This material is used by permission of Brooks/Cole, a Division of Cengage Learning.*

also the angular "height" of the star as seen by the observer (Abell, 1975; p. 115).

The *azimuth* is the number of degrees along the horizon to the vertical circle of the star from some reference point on the horizon. In astronomical tradition, azimuth formerly was measured from the south point on the observer's horizon, but in modern practice azimuth is measured from the north point, in conformity with the convention of navigators and engineers (Figure 30.1). (However, as we shall see in the next section, physicists measure azimuth from the east.) In either case, azimuth is measured (from the north or south) to the east (clockwise to one looking down from the sky) along the horizon from 0 to 360°. The altitude and azimuth system is called the *horizon system* (Abell, 1975; p. 115). Latitude and longitude, permanently attached to the earth, are a horizon system.

We now turn to the definition of the *hour angle*. The measurement of time is based on the rotation of the earth. As the earth turns, objects in the sky appear to move around us, crossing a local meridian each day. Time is determined by the position in the sky, with respect to the local meridian, of some reference object on the celestial sphere. The interval between successive meridian crossings or *transits* of that object is defined as a *day*. The actual length of a day depends on the reference object chosen. Several different kinds of days, corresponding to different reference objects, are defined. Each kind of day is divided into 24 equal parts, called *hours* (Abell, 1975; pp. 130–131).

Time is reckoned by the angular distance around the sky that the reference object has moved since it last crossed the meridian. The motion of that point around the sky is like the motion of the hour hand on a 24-h clock. The angle measured to the west along the celestial equator from the local meridian to the *hour circle* passing through any object (for example, a star) is that object's *hour angle*. An hour circle is a great circle on the celestial sphere running north and south through the celestial poles. *Time* can be defined as the *hour angle of the reference object* (Abell, 1975; p. 131).

As an example, suppose that the star Rigel is chosen as the reference for time. Then when Rigel is on the meridian it is $0^h0^m0^s$, "Rigel time". Twelve Rigel hours later, Rigel is halfway around the sky, at an hour angle of 180°, and the Rigel time is $12^h0^m0^s$. When Rigel is only 1° east of the meridian, and one Rigel day is nearly gone, the star is at an hour angle of 359°, and the Rigel time is $23^h56^m0^s$ (Abell, 1975; p. 131).

Time can be represented graphically by means of a time diagram, as in Figure 30.2. Here we imagine ourselves looking straight down on the north celestial pole from outside the celestial sphere. The pole appears at a point in the middle of the diagram, and the celestial equator (not the earth's equator) appears as a circle centered on the pole. As the earth turns to the east, the local meridian of an observer on earth sweeps around the sky, so that its intersection with the celestial equator would move counterclockwise around the circle in the time diagram. However, it is customary to represent the observer's meridian as fixed, intersecting the celestial equator, say, at the top of the diagram. Then the celestial sphere

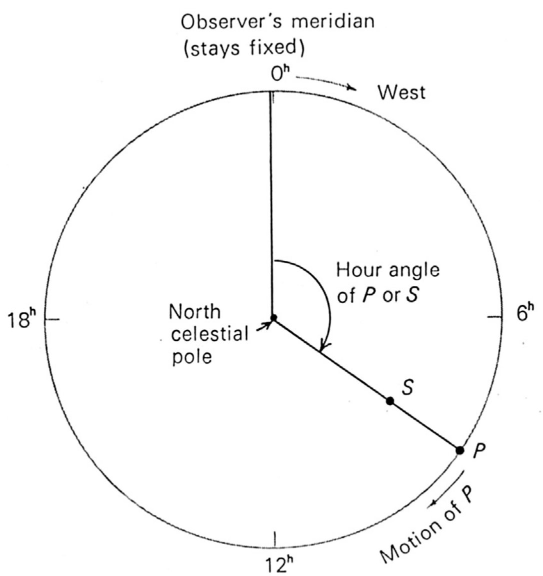

FIGURE 30.2 Time diagram. *From Abell (1975), p. 131. This material is used by permission of Brooks/Cole, a Division of Cengage Learning.*

must be regarded as rotating clockwise with respect to the meridian. Let the reference object be denoted by *S* (e.g., a star). Its (*S*'s) hour circle intersects the equator (great circle) at *P* (*P* for "point"), and as the celestial sphere rotates, the point *P* moves clockwise around the circle, like the hour hand of a clock. The hour angle of *S* (or *P*) increases uniformly with the rotation of the celestial sphere. In the diagram (Figure 30.2), it is shown as about 120°. The time would then be about 8^h (Abell, 1975; p. 131), as we shall see in the next paragraph.

Because of the relation between hour angle and time, it is often convenient to measure angles in time units. In this notation, 24 h corresponds to a full circle of 360°, 12 h to 180°, 6 h to 90°, and so on. One hour equals 15° and 1° is 4 min of time. Here we must distinguish between minutes and seconds of time (subdivisions of an hour), denoted by ^{m} and ^{s}, respectively, and minutes and seconds of *arc* (subdivisions of a degree), denoted by $'$ and $''$, respectively (Abell, 1975; p. 131). The conversion between units of time and arc is given in Table 30.1.

We can measure time by solar (Sun) time and by sidereal (star) time (Abell, 1975; pp. 132–133). *Sidereal* means "of the stars" (Webster's New World Dictionary of the American Language, 1959). Sidereal time is based on the sidereal day with its subdivisions of sidereal hours, minutes, and seconds. It is defined as the hour angle of the *vernal equinox*. The vernal equinox is defined as the point on the celestial sphere where the Sun in its apparent path around the sky (ecliptic) crosses the celestial equator from south to north. Another way to consider the vernal equinox is as follows. The north and south halves of the celestial sphere are separated by the celestial equator, halfway between the north and south celestial poles, and the place where the Sun crosses it on the first day of spring is called the vernal equinox (Abell, 1975; p. 30). It is one of the points on the celestial sphere where the celestial equator and the ecliptic intersect (Abell, 1975;

TABLE 30.1 Conversion between Units of Time and Arc

Time Units	Arc Units
24^h	360°
1^h	15°
4^m	1°
1^m	15′
4^s	1′
1^s	15″

From Abell (1975), p. 132. This material is used by permission of Brooks/Cole, a Division of Cengage Learning.

p. 115). The sidereal day begins at sidereal noon, $0^h0^m0^s$ sidereal time, with the vernal equinox on the meridian (Abell, 1975; p. 133).

Vernal means "belonging to spring." In the northern hemisphere, the vernal equinox occurs on about March 21 and the *autumnal* (fall) *equinox* occurs on about September 23 (Abell, 1975; p. 124). On the date of the *summer solstice*, which is about June 22 in the northern hemisphere, the Sun shines down most directly upon the northern hemisphere of the earth and the Sun appears 23½° *north* of the equator. (We shall see later why the angle is 23½° is important.) To a person at a latitude 23½° N, the Sun is directly overhead, and this latitude on the earth, at which the Sun can appear at the zenith at noon on the first day of summer, is called the *Tropic of Cancer*. The situation is reversed six months later, about December 22, the date of the *winter solstice*, which occurs about December 22 in the northern hemisphere. At latitude 23½° S, the *Tropic of Capricorn*, the Sun passes through the zenith at noon. It is winter in the northern hemisphere, summer in the southern (Abell, 1975; pp. 122–124). As we shall discuss later, the plane of the earth's equator is inclined at about 23½° (23°27′) to the plane of the ecliptic.

Sidereal time is useful in astronomy and navigation. The common coordinate system used to denote positions of stars and planets on the celestial sphere is referred to the celestial equator and the vernal equinox, much as latitude and longitude on the earth are referred to the earth's equator and meridian of Greenwich, England. Therefore, the position of a star in the sky with respect to the observer's meridian is directly related to the sidereal time. Every observatory maintains clocks that keep accurate sidereal time (Abell, 1975; p. 133).

However, sidereal time is not useful for daily living. We regulate our day by the Sun, not the vernal equinox (Abell, 1975; p. 133). The solar day is the period of the earth's rotation with respect to the Sun. As noted, the sidereal day is the time required for the earth to make a complete rotation with respect to a point in space, the vernal equinox (Abell, 1975; p. 132). A solar day is slightly longer than a sidereal day, as a study of Figure 30.3 shows. Suppose we start a day when the earth is at *A*, with the Sun on the meridian of an observer at point *O* on the earth. The direction from the earth to the Sun, *AS*, if extended, points in the direction *C* among the stars on the celestial sphere. After the earth has made one rotation with respect to the stars, the same stars in direction *C* will again be on the local meridian to the observer at *O*. However, because the earth has moved from *A* to *B* in its orbit about the Sun during its rotation, the Sun has not yet returned to the meridian of the observer but is still slightly to the east. The vernal equinox is so nearly fixed among the stars that the earth has completed, essentially, one *sidereal day*, but to complete a *solar day* it must turn a little more to bring the Sun back to the meridian. In other words, a solar day is slightly *longer* than a sidereal day, or one complete rotation

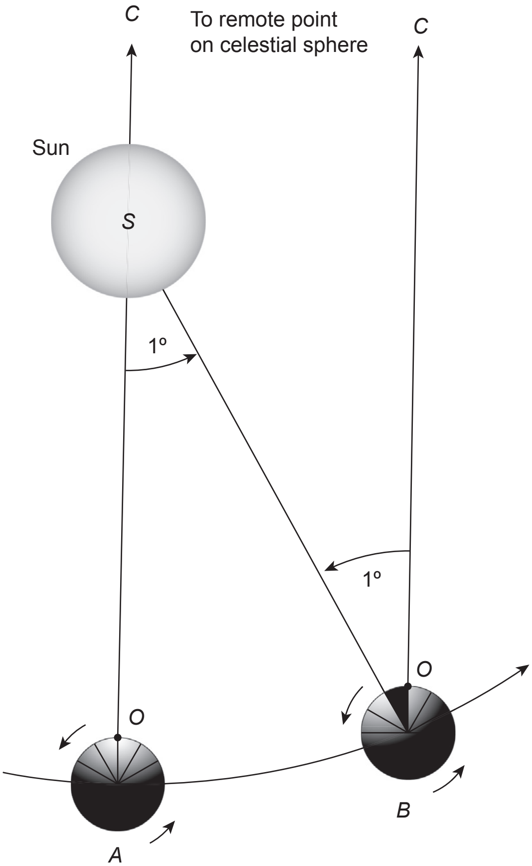

FIGURE 30.3 Sidereal and solar day. *From Abell (1975), p. 132. This material is used by permission of Brooks/Cole, a Division of Cengage Learning.*

of the earth. There are about 365 days in a year and 360° in a circle; thus the daily motion of the earth in its orbit is about 1° in relation to the Sun. This 1° angle, *ASB* (Figure 30.3), is nearly the same as the additional angle over and above 360° through which the earth must turn to complete a solar day. It takes the earth about 4 min to turn through 1°. A solar day, therefore, is about 4 min longer than a sidereal day (Abell, 1975; pp. 132–133).

On about September 23, the Sun passes through the autumnal equinox, halfway around the sky from the vernal equinox. On that date, at midnight, when the day begins, the vernal equinox is on the meridian, and so solar time and sidereal time are in agreement. With each succeeding day, however, sidereal time gains $3^{m}56^{s}$ on solar time, and the two kinds of time do not agree again until the daily difference between them accumulates to a full 24 h one year later (Abell, 1975; p. 133).

Sidereal time is reckoned by the hour angle of the vernal equinox, and *apparent solar time* is determined by the hour angle of the Sun. At midday, apparent solar time, the Sun is on the meridian. The hour angle of the Sun is the time *past midday* (*post meridiem* or P.M.). It is convenient to start the day not at noon, but at midnight. Therefore, the elapsed apparent solar time since the beginning of a day is the hour angle of the Sun plus 12 h. During the first half of the day, the Sun has not yet reached the meridian. We designate those hours as *before midday* (*ante meridiem*, or A.M.). We customarily start numbering the hours after noon over again, and designate them by P.M. to distinguish them from the morning hours (A.M.). But it is often useful to number the hours from 0 to 24, starting from the beginning of the day at midnight. For example, in various conventions, 7:46 P.M. may be written as $19^{h}46^{m}$, 19:46, or simply 1946 (Abell, 1975; p. 133). *Apparent solar time*, defined as the hour angle of the Sun plus 12 h, is the most obvious and direct kind of solar time. It is the time that is kept by a sundial. In a sundial, the raised marker, or *gnomon*, casts a shadow whose direction indicates the hour angle of the Sun. Apparent solar time was the time kept by man through many centuries (Abell, 1975; p. 133).

The exact length of an *apparent solar day*, however, varies slightly during the year. Recall that the difference between an apparent solar day and a sidereal day, if time is counted from noon on one day, is the extra time required, after one rotation of the earth with respect to the vernal equinox to bring the Sun back to the meridian. The length of this extra time depends on how far east of the meridian the Sun is after the completion of one sidereal day. The earth rotates to the east at a nearly constant rate of 1° every 4 sidereal minutes. Thus, if the Sun were exactly 1° east of the meridian, about four sidereal minutes would be needed to bring it the rest of the way to the meridian (Abell, 1975; p. 134).

The length of the apparent solar day would be constant, if the eastward progress of the Sun, in its apparent annual journey around the sky, were precisely constant. However, there are two reasons why the amount by which the Sun shifts to the east is not the same every day of the year. The first reason is that the earth's orbital speed varies. In accordance with Kepler's second law—the law of areas—the earth moves fastest when it is nearest the Sun (perihelion) in early January and slowest when it is farthest from the Sun (aphelion) in July. (For Kepler's three laws of planetary motion and his biography, see the Appendix, Section 30.4.) However, the earth's rate of rotation is nearly constant. Consequently, it (the earth) moves farther in its orbit during a sidereal day in January than in July (Figure 30.4). The Sun's apparent motion along the ecliptic is just the reflection of the earth's revolution, so the Sun's daily progress to the east reflects the inequalities of the earth's daily progress in its orbit. We see, then, that the extra amount by which the earth must turn after a

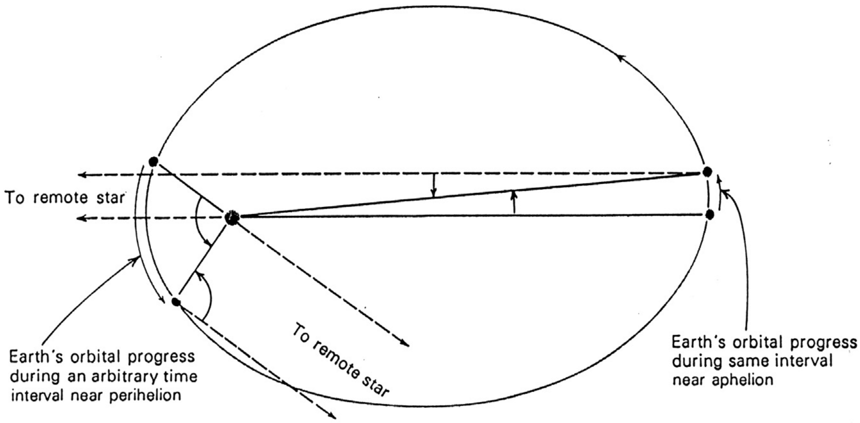

FIGURE 30.4 Variation in the length of an apparent solar day because of the earth's variable orbital speed. (The effect, of course, is grossly exaggerated.) *From Abell (1975), p. 134. This material is used by permission of Brooks/Cole, a Division of Cengage Learning.*

sidereal day to complete a rotation with respect to the Sun is not always exactly the same (Abell, 1975; p. 134).

The second reason for the variation in the rate of the Sun's eastward progress, and the consequent nonuniformity in the length of the apparent solar day, is that the Sun's path—the ecliptic—does not run exactly east and west in the sky, along the celestial equator, but is inclined to the equator by 23½°. [Globes of the earth are usually mounted with the earth's axis tilted from the vertical. This tilt is the same angle of 23½°, for that is the angle the earth's axis must make with the perpendicular to the plane of its orbit around the Sun (Abell, 1975; p. 121).] Even if the earth's orbit were circular, so that the Sun moved uniformly along the ecliptic, the amount by which it moved to the east would vary slightly throughout the year. The situation is illustrated in Figure 30.5, which shows the celestial sphere, the celestial equator, and the ecliptic. To make the effect more obvious, the obliquity of the ecliptic is exaggerated. Now suppose the Sun moved equal distances along the ecliptic near March 21 and June 22; such equal distances are marked off on the ecliptic in the figure. Near the vernal equinox, part of the Sun's motion is northward, and it progresses less to the *east* than it does along other points of the ecliptic. But, at the solstice, the Sun is moving not only due east, but it is also north of the equator where the hour circles converge, so that a 1° advance on the ecliptic is more than a 1° advance to the east. A similar analysis shows the Sun would also make more eastward progress near the winter solstice than near the autumnal equinox. With the actual 23½° obliquity, it turns out that a 1° advance on the ecliptic corresponds to 0.92° advance to the east at the equinoxes and 1.08° advance to the east at the solstices. Thus,

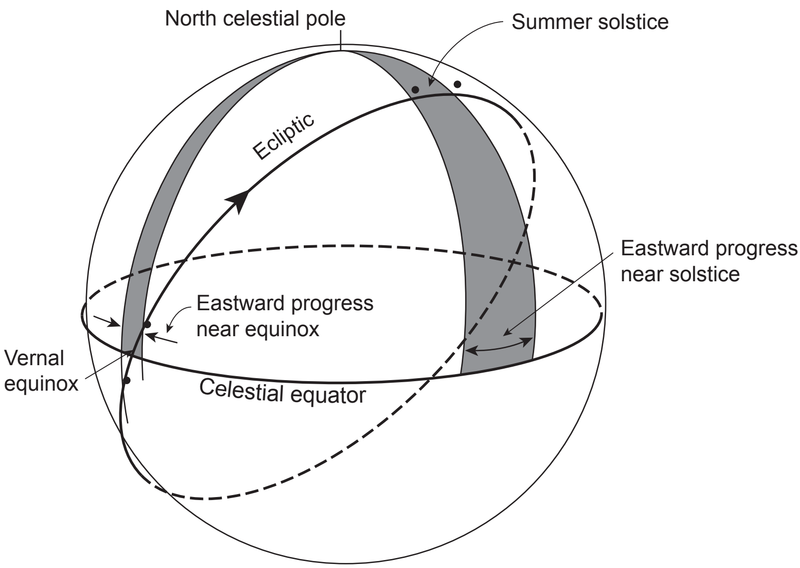

FIGURE 30.5 The Sun's apparent eastward daily progress varies because of the obliquity of the ecliptic. *From Abell (1975), p. 135. This material is used by permission of Brooks/Cole, a Division of Cengage Learning.*

even if the Sun did move uniformly on the ecliptic, its eastward progress would be variable (Abell, 1975; pp. 134–135).

The apparent solar day is always about 4 min longer than a sidereal day, but because of the Sun's variable progress to the east, the precise interval varies by up to one-half minute one way or the other. The variation can accumulate after a number of days to several minutes. After the invention of clocks that could run at a uniform rate, it became necessary to abandon the apparent solar day as the fundamental unit of time. Otherwise, all clocks would have to be adjusted to run at a different rate each day.

Mean solar time is based on the *mean solar day*, which has a duration equal to the *average* length of an apparent solar day. Mean solar time is defined as the hour angle of the mean Sun plus 12 h, where the *mean Sun* is a fictitious point along the celestial equator with the same average eastern rate as the true Sun. In other words, mean solar time is just apparent solar time averaged uniformly (Abell, 1975; p. 135).

The irregular rate of apparent solar time causes it to run alternately ahead of and behind mean solar time. The difference between the two kinds of time can accumulate to about 17 min. The difference between apparent solar time and mean solar time is called the *equation of time*, shown graphically in Figure 30.6. One can read from the plot, for any date of the year, the correction to apply to mean solar time to obtain apparent

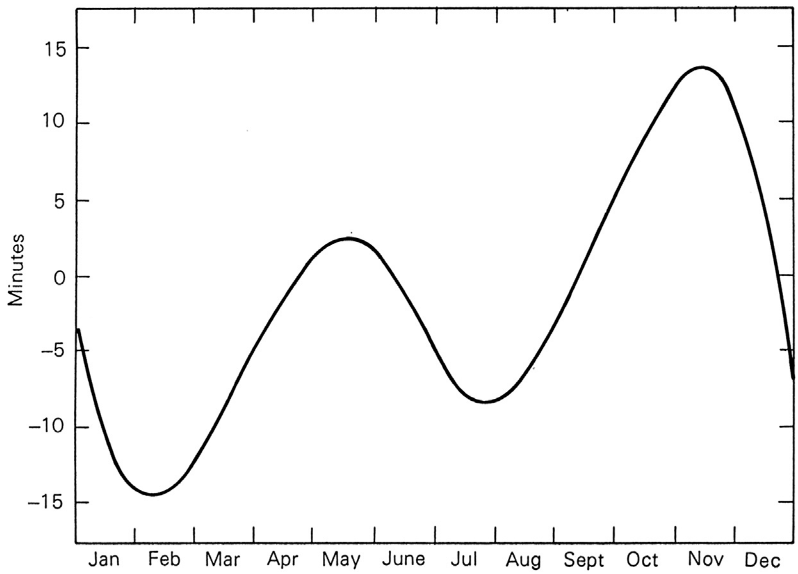

FIGURE 30.6 Equation of time (apparent minus mean solar time). *From Abell (1975), p. 136. This material is used by permission of Brooks/Cole, a Division of Cengage Learning.*

solar time. When the equation of time is positive, apparent time is ahead of mean time. Often the equation of time is plotted on globes of the earth as a nomogram, shaped like the figure eight and placed in the region of the South Pacific Ocean.

Although mean solar time has the advantage of progressing at a uniform rate, it is still inconvenient for practical use. Recall that it is defined as the hour angle of the mean Sun. But hour angle refers to the local celestial meridian, which is different for every longitude on earth. Thus, observers on different north–south lines on the earth have a different hour angle of the mean Sun and hence a different mean solar time. If mean solar time were strictly observed, people traveling east or west would have to reset their watches continually as their longitude changed, if it were always to read the local mean time correctly (Abell, 1975; p. 136).

Until the end of the nineteenth century, every city and town in the USA kept its own local mean time. With the development of railroads and telegraph, however, the need for standardization became necessary. In 1883, the nation was divided into four time zones (Abell, 1975; p. 136). Within each zone, all places keep the same time, the local mean solar time of a standard meridian running more or less through the middle of each zone. Now travelers reset their watches only when the time change has amounted to a full hour. In the USA for local convenience, the boundaries between the four time zones are chosen to correspond, as much as possible, to divisions between states. Mean solar time, so standardized, is called standard time. The standard time zones in the USA (not including Alaska and Hawaii) are Eastern Standard Time (EST), Central Standard Time (CST), Mountain Standard Time (MST), and Pacific Standard Time (PST), which, respectively, keep the mean times of the meridians of 75°, 90°, 105°, and 120° west longitude. Hawaii and Alaska both keep the time of the meridian 150° west longitude, 2 h less advanced than Pacific Standard Time (Abell, 1975; p. 136).

In 1884, an international conference was held in Washington, DC, in which 26 nations were represented (Abell, 1975; pp. 136–137). At that conference it was agreed to establish a system of 24 international time zones around the world. Each time zone, on the average, is 15° wide in longitude, although the zone divisions are usually irregular over land areas to follow international boundaries. At sea, the zone time of any place is the mean time of the standard meridian running through the center of the zone of that place. The zones are numbered consecutively from the Greenwich meridian. Those west of Greenwich are denoted (+) and those east are denoted (−). The EST time zone is zone number +5 (Abell, 1975; p. 137). Manhattan, Kansas, is zone number +6.

The procedure for determining standard time from apparent solar time, as read say, from a sundial, is illustrated in the following example (Abell, 1975; p. 139). At Los Angeles (118° west longitude), the apparent solar (sundial) time on March 16 was 11:30 a.m. From the equation of time (Figure 30.6), we note that on March 16 apparent solar time is 9 min behind mean solar time. Thus the local mean time is 11:39 a.m. Now Los Angeles is in the Pacific Standard Time zone, which keeps the time of the meridian at 120° west longitude. Los Angeles is 2° east of that meridian, so its local time is 8 min *more advanced* than that of the 120° meridian. Pacific Standard Time is thus 11:31 a.m.

30.2 INTERCEPTION OF DIRECT-BEAM SOLAR RADIATION

We now have an understanding of the Sun and its relation to earth to determine time. We need to know time to determine the how much of the Sun's energy is intercepted by plant leaves at each moment during the day. Because almost no information exists in textbooks demonstrating the interaction of leaf angles and Sun angles, the objective of this section is to describe Sun and leaf geometry to determine direct-beam solar radiation on a leaf at any time, angle, orientation, and location in the world. Direct-beam solar radiation is represented by I in the radiation-balance equation (Eqn (25.11)). It is the light that strikes us when the Sun is out and there are no clouds in the sky. We shall focus on the sloping surface of a plant leaf, but the procedure can be used for any inclined surface, including the sloping side of soil tilled up in a ridge.

To determine the light interception on a sloping surface, we need to know four angles. Two of the angles are the Sun's angles and two of the angles are the leaf's angles. For a theoretical consideration of these angles, see Kirkham (1986). Here we shall consider practical application of the theory.

Let us first look at the Sun's angles. Following along with what we learned in the previous section about altitude and azimuth (Figure 30.1), we can designate the Sun's position by two angles, *solar altitude* and *solar azimuth*. Solar altitude is the angle measured from the horizon up to the Sun. We shall abbreviate this angle A. It equals 0° when the Sun is on the horizon and 90° when the Sun is at the zenith directly overhead. The Sun is only directly above the earth, i.e., at 90° at solar noon, at two times. As noted in the previous section, these two times are about June 22 at the Tropic of Cancer, located at 23°27′ north of the equator, and about December 22 at the Tropic of Capricorn, located 23°27′ south of the equator. These times occur at the solstices. A solstice is defined as either of the two points on the Sun's apparent annual path where it is displaced farthest, north or south, from the earth's equator, that is, a point of greatest deviation of the ecliptic from the celestial equator (Glickman, 2000). An object would not cast a shadow if the Sun were 90° above it (i.e., at the Tropic of Cancer on about June 22 or the Tropic of Capricorn on about December 22). All other objects cast shadows when the Sun shines on them. In Manhattan, Kansas, during the entire year, the Sun is never directly above us (i.e., at an altitude of 90°). However at solar noon, it is at its zenith—the highest point the Sun rises during the day.

Solar azimuth is the angle measured clockwise from north (usually from north, but not always; see the next paragraph) to the projection of the Sun on the horizon. We shall abbreviate the solar azimuth with the Greek letter, lower case omega, ω. When measured from north, it equals 0° at north, 90° at east, 180° at south, 270° at west, and 360° at north to complete the full circle. Solar azimuth is measured from "true" or "geographic" north. True north is the direction toward the North Pole of the earth's axis. Although surveyors often use "magnetic" north, the direction toward which a magnetic compass points, solar designers must use true north, the basis of the Sun-angle charts. As a magnetic variation map shows (Vestine, 1971; Bennett, 1978, p. 7), the difference between magnetic and geographic north can be substantial. For example, in the USA it can be as much as 20° in Oregon.

Solar azimuth is not always measured from north, although it often is. It appears that meteorologists and architectural designers measure solar

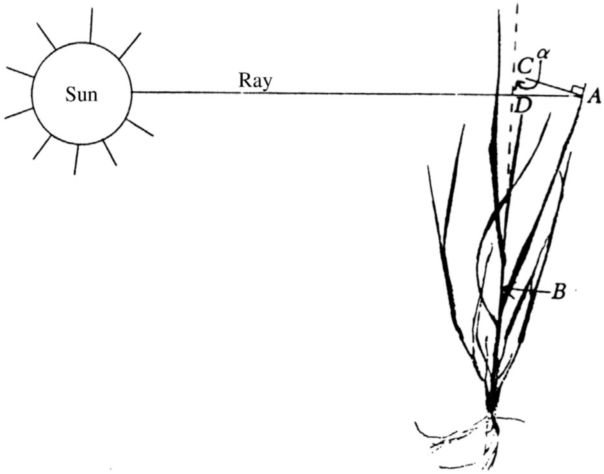

FIGURE 30.7 Solar ray on a wheat leaf. Angle ABC was measured. To calculate solar intensity on the leaf surface, the angle alpha (α) or angle ACB is needed. $\alpha = (90 - \text{measured angle})°$. The dotted line shows the extension of the vertical culm. The line CA is perpendicular to the leaf. *From Kirkham (1982), American Society of Agronomy. Reprinted by permission of the American Society of Agronomy.*

azimuth from north, such as Bennett (1978). East is sometimes used as the reference direction, especially by physicists. One needs to know from what direction (north or east) azimuth is measured.

Now let us turn to the two leaf angles that we need to know. The first angle we abbreviate with the Greek letter alpha, α, which is the "slope" angle of the leaf surface with respect to the vertical (Figure 30.7), and α is equal to 90° minus the slope measured from the horizontal. However, when we measure a leaf's angle (which we do with a protractor), we measure the angle between the vertical (e.g., stem or culm) and the leaf surface (angle ABC in Figure 30.7). So the measured angle is thus $(90-\alpha)°$ (Figure 30.7). The other leaf angle is called the aspect angle, which we abbreviate by the Greek letter beta, β. These aspect angles are shown in Table 30.2. For example, if the direction that the leaf points is north, then the direction that the leaf faces is south, and β is 180°.

Now that we know the four angles, we use the direct-beam solar radiation equation to determine the light interception on the sloping surface. The equation used to determine light intensity on inclined surfaces, like a plant leaf, has been published in several sources (Fons et al., 1960; Lee, 1963; ASHRAE, 1967, p. 475; Kondratyev, 1977). The equation has been in the literature for hundreds of years. My father, Don Kirkham, sought the origin of the equation. He traced it back to the 1700s, but could trace it no further. So the first scientist to derive the equation is apparently

TABLE 30.2 Relation between Leaf Orientation and the Angle β Used to Determine Direct-Beam Solar Radiation on a Leaf

Direction Leaf Points	Direction Leaf Faces	β, Degrees
N	S	180
S	N	0
E	W	270
W	E	90
NE	SW	225
NW	SE	135
SE	NW	315
SW	NE	45

From Kirkham (1986), Institute of Agrophysics, Polish Academy of Sciences, Lublin, Poland. Reproduced by permission of Prof. Dr. Jan Gliński, Editor-in-Chief, International Agrophysics, *Institute of Agrophysics.*

unknown (Don Kirkham, personal communication, c.1978). The form of the equation used here is given by Fons et al. (1960; see p. 4, Eqn 5 and p. 5, Eqn 8) as:

$$I/I_o = \sin A \cos \alpha - \cos A \sin \alpha \sin(Z - \beta), \tag{30.1}$$

where

I/I_o = intensity of received direct-beam radiation to incident direct-beam radiation (I' used by Fons et al. is I_o here)
A = solar altitude
$Z = \omega - 90°$, where ω is solar azimuth measured from north
α = angle between a vertical and a normal to the leaf; the angle measured with a protractor is between the leaf surface and the vertical (stem); the measured angle is therefore $(90-\alpha)°$, and $(90-\alpha)°$ is subtracted from 90° to get $\alpha°$
β = aspect angle of the leaf surface (direction leaf faces; N = 0°; NE = 45°; E = 90°, etc.) (Table 30.2)

Values of solar azimuth ω and solar altitude A are given by Bennett (1978).

Let us take an example, as given by Kirkham (1986). The measurement was taken in Stillwater, Oklahoma, on March 11, 1980. The second-to-the-top leaf of the winter wheat (*Triticum aestivum* L.) cultivar "Priboy" was measured. The leaf pointed southeast, and, therefore, faced northwest. The angle β was 315°. Angle α, the angle between the stem (culm) and the normal to the leaf, was 10°. The leaf was nearly horizontal. The measurement was taken at local solar noon, so the Sun was at its zenith; the azimuth angle ω was 180°, and $Z = 180° - 90° = 90°$. (To get local solar time, one's watch should read 12 noon when the Sun is at its maximum height for the day.) The solar altitude angle was 50° and it was obtained from Bennett (1978, p. 51). So $Z = 90°$; $\alpha = 10°$; $A = 50°$; $\beta = 315°$. Putting these values in Eqn (30.1), we get:

$$\begin{aligned} I/I_o &= \sin 50° \cos 10° - \cos 50° \sin 10° \sin(90 - 315)° \\ &= (0.766)(0.985) - (0.643)(0.174)(0.707) \\ &= 0.754 - 0.079 = 0.675. \end{aligned}$$

As a check, we can read a similar I/I_o value of 0.6873 on p. 77 of Fons et al. (1960).

Kirkham (1984) measured the interception of direct-beam solar radiation of two cultivars of winter wheat: 'KanKing', a drought-resistant cultivar, and 'Ponca', a drought-sensitive cultivar. KanKing as developed by Earl G. Clark, a Kansas farmer and wheat breeder, during the drought years of the 1950s in Kansas (Heyne, 1956). They were grown under well watered conditions. However, the drought-resistant cultivar had its leaves prostrate on the ground during the winter, while the drought-sensitive cultivar had erect leaves. The horizontal leaves of KanKing intercepted more direct-beam solar radiation than the erect leaves of Ponca (Figure 30.8). The leaves of KanKing were wider during the winter than those of Ponca, perhaps because it intercepted more direct-beam solar radiation during the winter than Ponca. KanKing's leaf orientation may be one reason why it is drought-tolerant. It can intercept more solar radiation and grow more during the winter, putting its roots down deeper and thus allowing a deeper root zone for water extraction

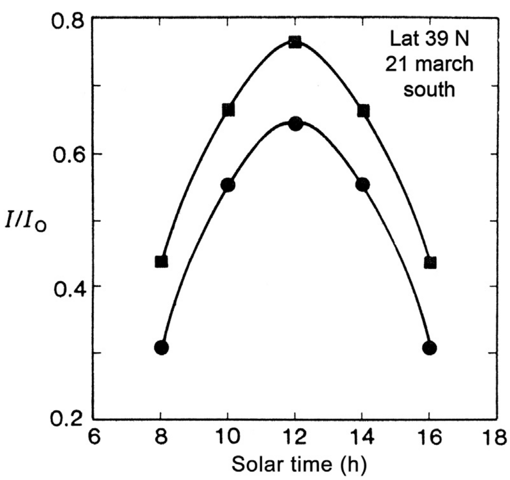

FIGURE 30.8 Calculated ratio of intensity of received direct-beam radiation to intensity of incident direct-beam radiation (I/I_o) for south-facing leaves of a winter wheat cultivars with horizontal leaves (e.g. 'KanKing') (squares) and a winter wheat cultivar with erect leaves (e.g. 'Ponca') (circles). *From Kirkham (1984), Crop Science Society of America. Reprinted by permission of the Crop Science Society of America.*

during the spring growing season. Kirkham (1984) did not measure the root depth of Ponca and KanKing, but the relation between I/I_0 and root depth needs to be determined.

We now have a way to quantify the amount of solar energy intercepted by plant leaves. Just as buildings and solar-powered installations can be placed to maximize interception of the Sun's rays, cultivars of different plants can be developed for improved efficiency of solar energy use (Kirkham and Smith, 1984). We could orient plant rows so that leaves would intercept more solar energy and, thereby, increase photosynthesis and plant production (Kirkham, 1982).

We have applied the direct-beam solar radiation equation to a plant leaf. For its application to the sloping surface of soil, see Affleck et al. (1976), who worked in Iowa, where it is important to orient seed beds so they can warm up early in the spring and corn (*Zea mays* L.) can get a good start on growth. If the corn is planted on the south-facing side of a sloped surface, it will warm up more than if it is planted on a flat surface.

30.3 HOW TO MEASURE ALTITUDE AND AZIMUTH ANGLES OF SUN

We can determine solar altitude and azimuth if we do not have tables or graphs, such as those in Bennett (1978). To use this method of getting altitude and azimuth, we do not need to know solar time. However, we do need tables of tangents and sines or a calculator that can calculate them. A freshman astronomy student might be more interested in this method than a crop scientist. However, the Sun and its angles are important in crop growth, so students of crop science also need to know the information. The procedure comes from Don Kirkham (personal communication, February 3, 1983) and is illustrated in Figure 30.9.

We have a vertical rod of height h that casts a shadow of length s. The rod could be a meter stick. We draw an East–West line through the end of the shadow. We can get the East–West line from a compass or know it from hedgerows, fences, highways, etc. The abbreviations in Figure 30.9 are as follows:

$L =$ length of the perpendicular from the E–W line to the bottom of the rod; L is the shortest walking distance from the bottom of the rod to the E–W line.
$x =$ projection of a point at the bottom of the rod on the E–W line, which goes through the end of the shadow of the rod
$\alpha =$ altitude (angle) of the Sun
$\omega =$ azimuth (angle) of Sun measured from east

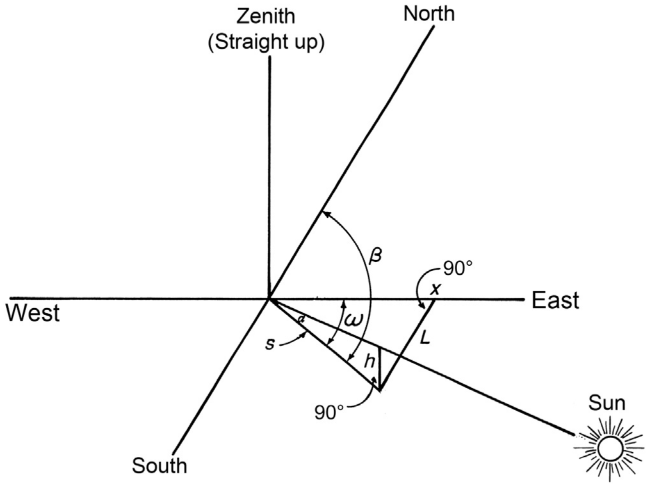

FIGURE 30.9 Illustration of how to measure altitude and azimuth angles of the Sun. See text, Section 30.3, for explanation of the symbols. *Figure drawn by Don Kirkham, Iowa State University, Ames, Iowa.*

$\beta = \omega + 90° =$ azimuth (angle) of Sun measured from north
$\tan \alpha = h/s$
$\alpha = \tan^{-1} h/s$
$\sin \omega = L/s$
$\omega = \sin^{-1} L/s$

While the meter stick is in the vertical position, the shadow could be scratched in the soil. Then the length of the scratch could be measured with the meter stick. After drawing an East–West line through the end (furthest from the rod) of the shadow, one could use the meter stick to measure L.

Example:

$h = 100$ cm
$s = 99$ cm
$L = 95$ cm
$\tan \alpha = h/s = 100/99$
$\alpha = \tan^{-1} 100/99$
$\alpha = 45.3°$ or $45°$ (altitude angle of Sun)
$\sin \omega = L/s = 95/99$
$\omega = \sin^{-1} 95/99$
$\omega = 74°$ (azimuth angle of Sun measured from east)
$74° + 90° = 164°$ (azimuth angle of Sun measured from north).

Bennett (1978, p. 6) gives a method of determining true north. When beginning work at a specific site, a designer can read true north from a map, such as those drawn by the United States Geological Survey, or determine it through the following procedure. Developed by Vitruvius (Roman architect and engineer of the first century BC), the process relies upon the fact that the Sun's motion is symmetrical around true south. On a sunny day, do the following:

1. In the morning, place a stake in a clearing on the site.
2. Mark the end of its shadow.
3. With the shadow as the radius, draw a circle on the ground around the stake.
4. In the afternoon, when the end of the shadow again falls exactly on the circle, mark its end point a second time.
5. Divide in half the distance between the two marks (that made in the morning and that in the afternoon). A line from the stake to this spot points due north.

30.4 APPENDIX: BIOGRAPHY OF JOHANNES KEPLER

Unless noted, all the following biographical material on Kepler comes from Ronan (1971).

Johannes Kepler (1571–1630) was a German astronomer whose studies of the motions of the planets helped to lay the foundations of modern astronomy. He was born on December 27, 1571, at Weil in Württemberg. His father was a petty officer in the duke of Württemberg's army and his mother, Catherine (née Guldenmann), came from a family of once noble standing. Johannes was born premature by two months and was a delicate child. With his younger brother Heinrich, he lived with his grandparents while his father was engaged in the Dutch wars and his mother accompanied her husband to the Netherlands. When about four years of age, he contracted small-pox and nearly went blind with the result that his eyesight was permanently impaired.

He first attended school in Weil, then in Leonberg on the return of his parents, and finally in the convent schools of Adelberg (1584) and Maulbronn (1586). In September, 1588, he obtained his bachelor's degree and entered the University of Tübingen the following year. There, in August 1591, he obtained his master's degree. He had hoped to enter the ministry but instead was persuaded to accept the post of professor of mathematics at Graz in 1594.

At Tübingen, Kepler came under the influence of Michael Maestlin (an astronomer) who became a lifelong friend. Maestlin was a protagonist of the views of Copernicus, which he had to teach privately to his young pupil, because the Ptolemaic system was still the picture of the universe that was held in official circles. Kepler absorbed the heliocentric concept of Copernicus and later developed it with brilliance. [We remember that the Ptolemaic system, named after Ptolemy, an Alexandrian astronomer, mathematician, and geographer, who lived in the second century AD, held that the earth was the center of the universe, around which the heavenly bodies moved. The Copernican system was the theory of Nicolaus Copernicus, a Polish astronomer (1473−1543), which held that the planets revolve around the Sun and that the turning of the earth on its axis accounts for the apparent rising and setting of the stars. The Copernican system is the basis of modern astronomy (Webster's New World Dictionary of the American Language, 1959).]

As professor at Graz, Kepler lectured on mathematics and also, at the request of the university, on Virgil (Roman poet; 70−19 BC) and rhetoric. He was also expected to prepare annual almanacs giving astronomical and astrological predictions. Kepler, was, however, primarily interested in the problems of the planetary system. He believed that there must be some regularity in the relationship between the orbits of the planets, and he spent his major efforts in seeking this relationship. The first fruits of his labors were published in 1596 under the title *Prodromus Dissertationum Mathematicarum continens Mysterium Cosmographicum*. The publication of this work brought him fame, and he began corresponding on friendly terms with Tycho Brahe (1546−1601), a Danish astronomer, and Galileo Galilei, known as Galileo (1564−1642), an Italian astronomer and physicist, two of the most eminent astronomers of the day.

In April of 1597, Kepler married the wealthy Barbara von Mühleck, whom he had met in Graz, but by September all Protestant theologians were expelled from the city. He left, but his wife's influence allowed him to return after a month. However, the situation remained unfavorable and in 1600 Kepler and his wife had to leave Graz. The Roman Catholic Hensart von Hohenberg, who was an old pupil of Maestlin's and with whom Kepler had been in correspondence, suggested that Tycho Brahe might help. Tycho himself had been forced to leave the island of Hveen, but when he was appointed court mathematician to Emperor Rudolf II in Prague, he invited Kepler to join him there, and Kepler and his wife made their home in Prague.

Tycho died a year after Kepler's arrival in Prague, and he was thereupon appointed by the emperor to succeed his late patron as court mathematician, although at a reduced salary. All the observations which Tycho had made were at Kepler's disposal, and as later events proved, these were a veritable storehouse of information to which astronomy will ever be in debt.

In 1604, he published *Astronomiae pars optica*. This book contained Kepler's fundamental ideas on the nature of vision and a definition of a ray of light, which later became generally adopted in geometrical optics. He also offered an explanation of the reflection of light and an approximation to the law of refraction. (It is interesting to note that a man with limited eyesight made these important contributions to the understanding of vision and light.)

Unlike Tycho, Kepler had no talent for experimentation (Cajori, 1929; p. 33). But he was a great thinker and excelled as a mathematician. He absorbed Copernican ideas and early grappled with the problem of determining the real paths of the planets. He studied observations on the planets recorded by his master. He took the planet Mars and found that no combinations of circles would give a path which could be reconciled with the actual observations. In one case the difference between the observed and his computed values was 8 min, and he knew that an accurate observer like Tycho could not make such an error so great. He tried an ellipse for the orbit of Mars, and it fitted. Thus, after more than four years of assiduous computation, and after trying 19 imaginary paths and rejecting each because it was more or less inconsistent with observation (Cajori, 1929; p. 34), Kepler at last discovered that the planets revolved round the Sun in elliptical paths with the Sun at one focus, and this formed his first law. It was his efforts to reconcile the observations with the Copernican system that Kepler was led to take this bold step, making a complete break with the traditions of more than 2000 years, and it had a profound effect on future astronomy.

Next, with his belief in the order and regularity of the heavens, he sought some kind of regular motion which would describe the behavior of the planets in paths of elliptical form. Such work entailed much labor, but finally he discovered that the radius vector of each planet described equal areas of the ellipse in equal times (his second law). This new and fundamentally important work, the *Astronomia nova*, was published in Prague in 1609, and the book made public his two laws to date of planetary motion.

In 1612, he was appointed mathematician to the states of upper Austria. In 1613, with a new emperor, he advocated the introduction of the Gregorian calendar, but was frustrated by antipapal prejudice. During this year, he married Susanna Reutlinger, his first wife having died two years earlier.

Kepler extended his planetary laws to the satellites of Jupiter in the *Epitome astronomiae Copernicanae* published in parts between 1618 and 1620−1621. He published his third planetary law connecting the periods and mean distances of the planets in 1619 in his *De Harmonice Mundi* with which his great contributions to astronomy were completed.

In sum, Kepler's three laws are as follows (Cajori, 1929; p. 34): (1) Each planet moves in an ellipse, having the Sun in one of its foci; (2) The radius vector joining the Sun with a planet moves over equal areas in equal times; (3) The square of the times of revolution of any two planets are proportional to the cubes of their mean distance from the Sun, i.e., $T^2 : T_1^2 = D^3 : D_1^3$. It was conjectured by Newton, and also by Hooke and others, that if Kepler's third law was true, then the attraction between the earth and other members of the solar system varied inversely as the square of the distance. The accuracy of Kepler's third law was doubted at that time. To show that the above conjecture was true required the genius of Newton (Cajori, 1929; p. 64).

In 1628, Kepler and his family moved to Silesia, where he continued to work. On November 15, 1630, in Ratisbon (Regensburg), Kepler died while visiting a meeting of imperial electors (Voelkel, 1999; p. 135), leaving behind him achievements that changed astronomy and ensured his permanent fame.

References

Abell, G.O., 1975. Exploration of the Universe, third ed. Holt, Rinehart, and Winston, New York. 738 pages.

Affleck, S.B., Kirkham, D., Buchele, W.F., 1976. Seedbed preparation for optimum temperature, moisture, aeration and mechanical impedance. In: Proceedings of the 7th Conference of the International Soil Tillage Research Organization, ISTRO. Report No. 45. Division of Soil Management, Agricultural College of Sweden, Uppsala, Sweden. S-750 07.

ASHRAE, 1967. Handbook of Fundamentals. American Society of Heating, Refrigerating and Air-Conditioning Engineers, New York, NY, 544 pp.

Bennett, R., 1978. Sun Angles for Design. Robert Bennett Architect and Engineer, Bala Cynwyd, Pennsylvania, 76 pp.

Cajori, F., 1929. A History of Physics. Macmillan, New York.

Fons, W.L., Bruce, H.D., McMasters, A., 1960. Tables for Estimating Direct Beam Solar Irradiation on Slopes at 30°−46° Latitude. Pacific Southwest Forest and Range Experiment Station, Forest Service, United States Department of Agriculture, Berkeley, California, 298 pp.

Glickman, T.S., 2000. Glossary of Meteorology, second ed. American Meteorological Society, Boston, Massachusetts. 855 pp.

Heyne, E.G., 1956. Earl G. Clark, Kansas farmer wheat breeder. Trans. Kans. Acad. Sci. 59, 391−404.

Kirkham, M.B., 1982. Orientation of leaves of winter wheat planted in north-south or east-west rows. Agron. J. 74, 893−898.

Kirkham, M.B., 1984. Leaf orientation, light reception, and growth of winter wheat. Crop Sci. 24, 925−928.

Kirkham, M.B., 1986. Theoretical consideration of direct-beam solar radiation on plants leaves. Int. Agrophys. 2, 53−58.

Kirkham, M.B., Smith, E.L., 1984. Solar intensity on winter wheat leaves. Field Crop. Res. 8, 297−306.

Kondratyev, K. Ya, 1977. Radiation Regime of Inclined Surfaces. Tech. Note 152, WMO No. 467. World Meteorological Organization, Geneva, 82 pp.

Lee, R., 1963. Evaluation of Solar Beam Irradiation as a Climatic Parameter of Mountain Watersheds. Colorado State University Hydrology Paper 2, Fort Collins, Colorado, 50 pp.

Ronan, C.A., 1971. Kepler, Johannes. Encyclopaedia Britannica 13, 309−311.

Sobel, D., 2007. Longitude. The True Story of a Lone Genius Who Solved the Greatest Scientific Problem of His Time. Walker Pub. Co, New York.

Vestine, E.H., 1971. Geomagnetism. Encyclopaedia Britannica 10, 179−185.

Voelkel, J.R., 1999. Johannes Kepler and the New Astronomy. Oxford University Press, New York and Oxford.

Webster's New World Dictionary of the American Language, college ed., 1959. World Publishing, Cleveland.